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Analytical Solution of a Non-isothermal Flow in Cylindrical Geometry

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Abstract

The work puts forth an analytical solution to the cylindrical domain transport problem in a Newtonian fluid. It is assumed that the flow is axi-symmetric and that it is dominated along the channel axis. While temperature and concentration have Dirichlet boundary values, velocity is subject to a no-slip boundary constraint. The resultant problem is converted into a set of differential equations with non-trivial variable coefficients in a cylindrical shape. The closed-form analytical solutions are generated for the flow variables by using Frobenius' series solution approach. After analysing the generated model, we were able to demonstrate that the flow variables are axi-symmetric. In order to demonstrate that the obtained concentration model is positivity preserving—that is, it produces positive concentration—we additionally declare and demonstrate a further theorem, assuming that the boundary value is non-negative. Lastly, we talk about the impact of the pertinent flow parameters and show graphical findings for the flow variables. The findings demonstrated that (i) the temperature falls as the cooling parameter increases (), (ii) the fluid velocity decreases with an increase in the cooling parameter (), and (iii) an increase in the injection parameter () results in an increase in the concentration.

Keywords: Channel flows, temperature, concentration, velocity, and non-isothermal conditions.

Introduction

There are many uses for the study of flows in channels in both science and technology. These include the arteries and blood vessels in humans, the flow of water and oil through reservoirs, and the use of chemical engineering for filtration.

and purification procedure, the flow of natural gas, and river flows along cylinder-shaped channels. Thus, knowing how channel flows works is quite useful.

This has sparked a lot of research in the field. Jahangiri et al. [1] investigated, for example, how non-Newtonian blood behaviour affected wall shear stress in an elastic artery exhibiting both simple and sequential stenosis. Their numerical solution to the problem demonstrated that the power law model is unsuitable for modelling the non-Newtonian behaviour of blood. A mathematical model for the blood flow via an overlapping stenosed artery with a core region under the influence of a magnetic field was created in the same vain by Lokendra et al. [2]. The issue was resolved analytically, and the outcome demonstrated that the application of an external magnetic field may be used to regulate blood velocity and shear stress on the arterial wall caused by overlapping stenosis. However, heat and mass were not taken into account in this research. Lukendra et al.'s study [3] examined the pulsatile blood flow in an inclined tapering artery with minor stenosis via a porous medium with constant permeability. The flow field, wall shear stress, volumetric flow rate, and effective viscosity are all significantly influenced by the magnetic field, velocity slip, inclination, and permeability of the porous medium, according to the analytical solution of their fluid problem. Heat and mass were not examined in their work. Furthermore, Rathod and Shakera [4] took into account the pulsatile blood flow via a porous media. After solving analytical problems, they came to the conclusion that, while velocity distribution decreases as the magnetic parameter increases, it grows as body acceleration and porous medium permeability increase. Heat and mass transmission were not taken into account. The pulsatile flow of blood through a stenosed artery under the impact of external periodic body acceleration was studied by Nagarani and Sarojamme [5]. The issue was analytically resolved, and the outcome shown that stenosis and yield stress both lower flow rates and raise flow resistance. Heat and mass transmission were not taken into account in their calculation.

These flows occur in cylindrical domains, such as the human blood artery, and are similar to those in the cardiovascular system of humans. Compared to models in Cartesian coordinate systems, fluid dynamics models in these domains are more difficult. When mass and heat transfer are included in the system, the issue becomes more challenging.

In [6], for example, two finite difference techniques for channel flow issues were explored. Their findings showed that, for values of a model parameter less than the theoretically calculated bound, no numerical oscillations were seen. Similarly, [7] examined the theoretical study and applications of a convergent numerical approach to the convective channel flow heat

transfer problem. Using a numerical scheme, they were able to demonstrate that while increasing the Brinkman number increases temperature, increasing the pressure gradient increases velocity. The flow of an electrically conducting and radiating fluid across a moving, heated porous plate in the presence of an induced magnetic field is shown in a different dimension in [8]. The results of their analytical solution, which used the perturbation approach, demonstrated that suction had a greater impact on the magnetic field and fluid velocity but less on temperature.

Furthermore, [9] took into account thermal radiation, cross-diffusion, and continuous suction while analysing the flow of a fluid with changing viscosity that involves heat and mass transfer. Therefore, the governing partial differential equations (PDEs) are transformed into ordinary differential equations by the use of variables, which are then solved using Runge-Kutta techniques. According to their findings, the fluid velocity rises as the Dufour and Soret parameters are increased. They also emphasised that the viscosity changes linearly with temperature, the flow is only steady, and the suction is constant.

Blood oscillation in a stenosed artery under the effect of a transverse magnetic field through a porous media was investigated by Jain et al. [10]. They came up with the Bessel answer after solving analytically. [11] examined the characteristics of blood flow across a mildly stenotic, sloping, tapering porous artery. The analytical solution to the problem yielded the Bessel solution.

The boundary layer flow in a spinning MHD fluid was studied by Amos et al. [12]. The outcome demonstrates that while increasing heat radiation causes a drop in flow velocity, increasing magnetic, Schmidt number, chemical reaction, and rotation parameter increases flow velocity in the system. While increasing the mass transmission, the chemical reaction parameter reduces the heat transfer. [13] examined, assuming a long wavelength and low Reynolds number, the MHD peristaltic motion of a third grade fluid in an asymmetric channel. When Deborah number is small, series solutions of axial velocity and pressure gradient were obtained using perturbation approach. Wall stretching and magnetic field with channel in the work of [14]

The copper-kerosene nanofluid flow was investigated. Heat transfer from natural convection with heat sources in the cylindrical domain was carefully considered by [15]. A different scenario uses a different foundation in the channel flow to take into account the properties of thermal Titania nanofluids in a linked system of Maxwell convective heat under the Navier-Strokes equation. [16] examined the radiating fluid flow in heated porous materials. The findings demonstrated that suction is more effective when the fluid velocity is high, and magnetic field ineffective when the temperature profile is high.

As was already noted, studying fluid flow in cylindrical geometry is a challenging task that becomes more difficult when mass flow and heat are included. When an analytical solution to the problem is attempted, the difficulty of the situation gets worse. The main goal of this work is to provide an analytical solution for the heat and mass flow problem in an incompressible fluid flowing through a cylindrical channel. The uniqueness of the generated fluid models at the origin, the channel's (tube's) centre, makes analytical solution derivation difficult.Applying the zero-gradient boundary condition at the origin becomes challenging as a result. This is the issue that was unsolvable in [17], which resulted in a solution that did not meet the channel wall's no-slip boundary requirement. In this study, we overcome this challenge by enforcing the restriction that all flow quantities or variables are limited across the channel. In the general solution of the flow variables, this eliminates the zero-Newmann boundary condition at the origin and enables us to set the infinite term to zero. In this manner, the intended outcome is achieved. This study's originality lies in this.

This is how the paper is presented: The problem's mathematical and physical models are presented in Section 2, and a thorough analytical solution is obtained in Section 3. In Section 4, we analyse the generated analytic solution by presenting a few Theorems with proofs. Section 5 presents and discusses the findings. Section 6 concludes the paper.

The goal of this work is to expand on the work of [11] by incorporating heat and mass transport and producing an analytical solution in closed form.

• Mathematical Formulation

It is assumed that the flow is axi-symmetric and that it is dominated along the channel axis. Velocity is subject to a no-slip boundary condition, whereas concentration and temperature have non-zero positive constants on the border. As indicated in Fig. 1, let r be the distance from the channel centre and u,T,C be the fluid velocity, → ⺄ 亖0, 0, u❽ 攵 temperature, and mass concentration. Assume that the fluid velocity is u~. r_e, ϱ_e, z_e
~r

Given the axisymmetric nature of the flow, let us consider the cylindrical channel coordinates. What is the radius?

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flow is maintained at a non-constant temperature (non-isothermal) along the horizontal axis z of the channel, whereas the vertical axis z represents the directions devoid of flow (see Fig. 1).

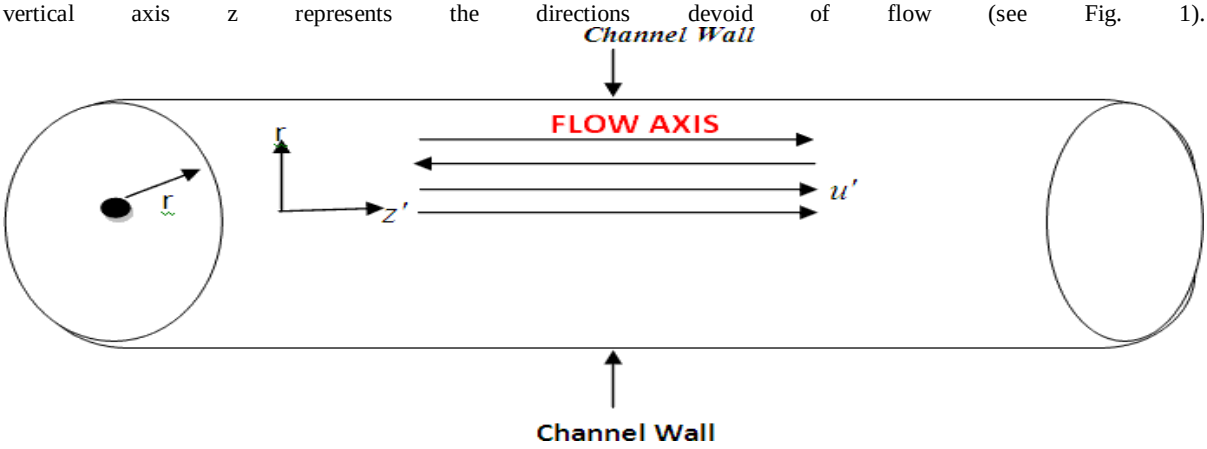

Fig. 1. Geometry of the physical model in horizontal channel

Based on the assumption of Boussinessq approximation of fluid model and taken consideration of induced magnetic field and steady flow; the equations governing the flow are:

• Results and Discussion

We have formulated and computed results of non-isothermal flow in a cylindrical channel. The parameter values used from the graphical presentations are shown as follows: However, the following parameter values were explicitly used in the simulation study:

- Vary velocity with magnetic field $\Pr = 4,$
- Vary velocity with $Gr : \mathrm{M} = 1.5,\ \mathrm{p} = 1,$

$Gr = 1.0,$
$\lambda = 1000$

$\lambda = 180.0$

- Vary velocity with $Pr : \mathrm{M} = 2.5,$

$Gr = 0.05,$

$\lambda = 188$

- Vary velocity with $\lambda : \mathrm{M} = 2.5,\ \Pr = 6,$
- Vary temperature with $\lambda : \mathrm{M} = \theta_a = 1.0$
- Vary temperature with $\theta_a : 1.0$
- Vary concentration with $\alpha : \varphi_a = 0.1$

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- Vary concentration with $\varphi_a : \alpha = 0.8$

$Gr = 0.05$

- **Results**

Therefore we present our results and discussion:

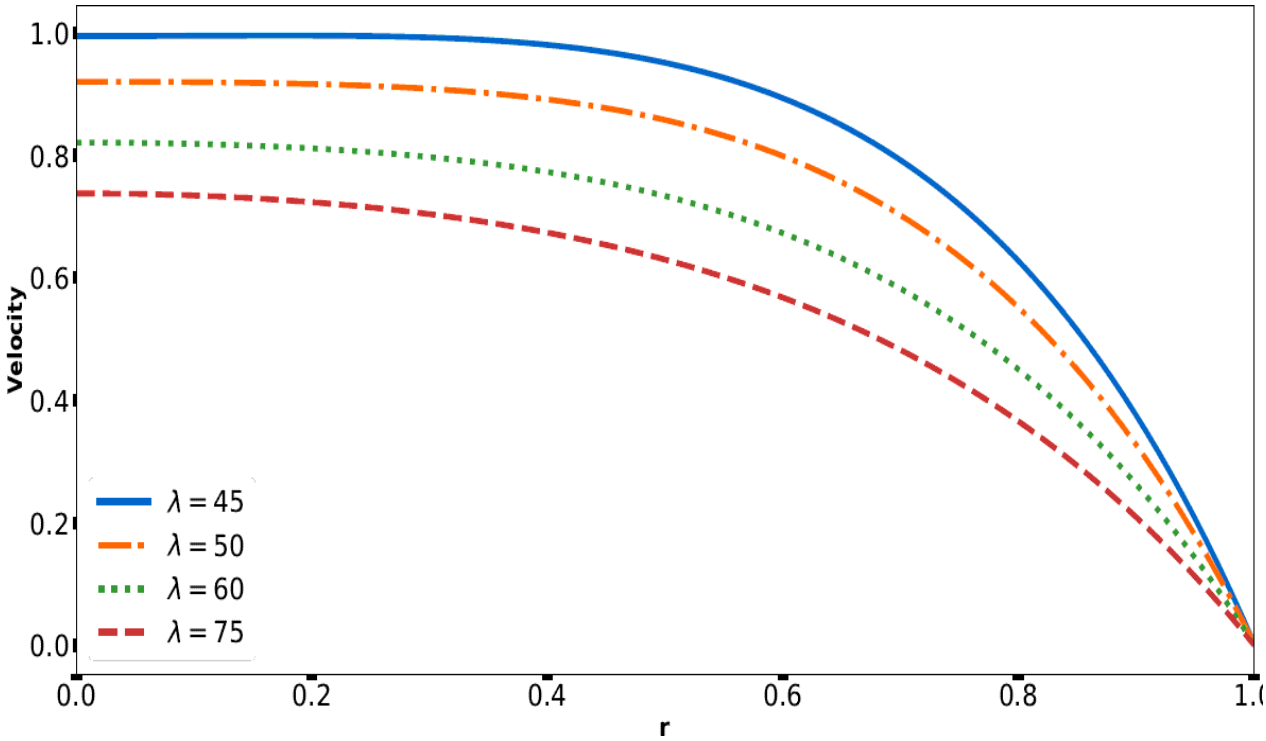

Fig. 2. Velocity profiles for $\Pr = 4$,

$Gr = 1.0$,

$\lambda = 180.0$ **and different values of velocity cooling**

parameter, λ

Fig. 2 shows the variation of fluid velocity with the cooling parameter. It can be seen that the velocity reduces with increasing cooling parameter. This is physically consistent because an increase in cooling parameter λ reduces fluid temperature which in turn in turn increases the fluid viscosity, hence reduces the fluid velocity. This result is in good agreement with the results of [8].

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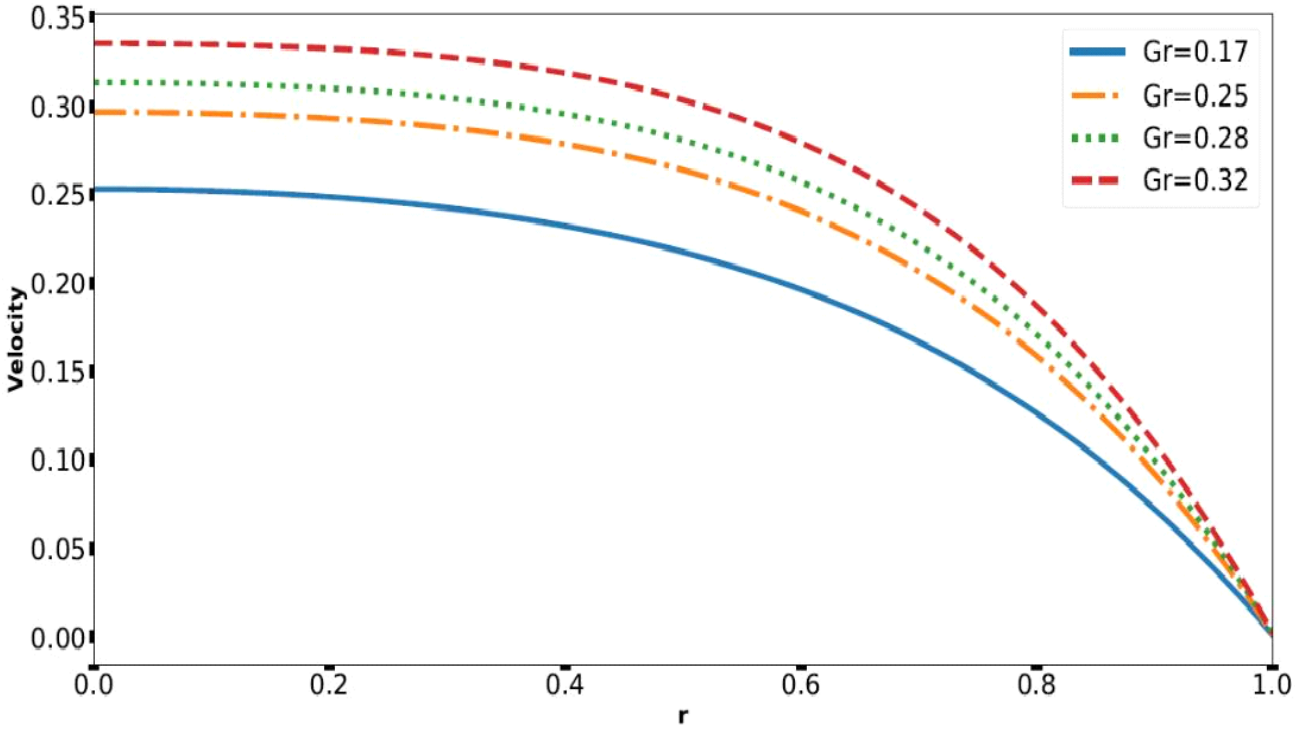

Fig. 3. Velocity profiles for $M = 1.5,\ p = 1,$

$\lambda = 1000$ **and different values of Grashof, number** Gr

Fig. 3 shows the variation of fluid velocity with Grashof number. It is clear that an increase in Grashof number leads to an increase in fluid velocity. This agrees with the results of [18] and [19].

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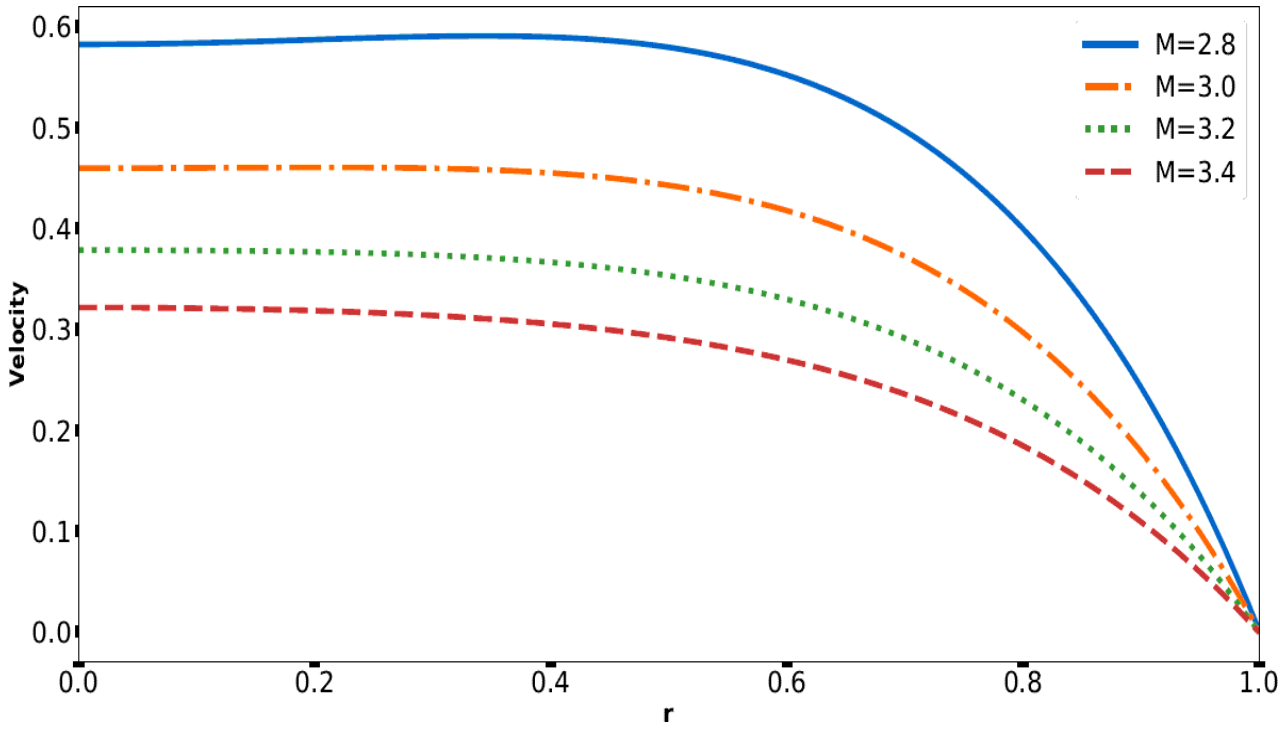

Fig. 4. Velocity profiles for $\mathrm{M} = 2.5,$

$Gr = 0.05,$

$\lambda = 188$ **and different values of Magnetic field**

parameter, M

Fig. 4 shows the variation of fluid velocity with magnetic field parameter. It can be seen that increase in the magnetic field parameter decreases the fluid velocity. This is in accordance with the results of [8,10,16,20,21,22].

Fig. 5. Velocity profiles for with $\mathrm{M} = 2.5,\ \mathrm{Pr} = 6,$

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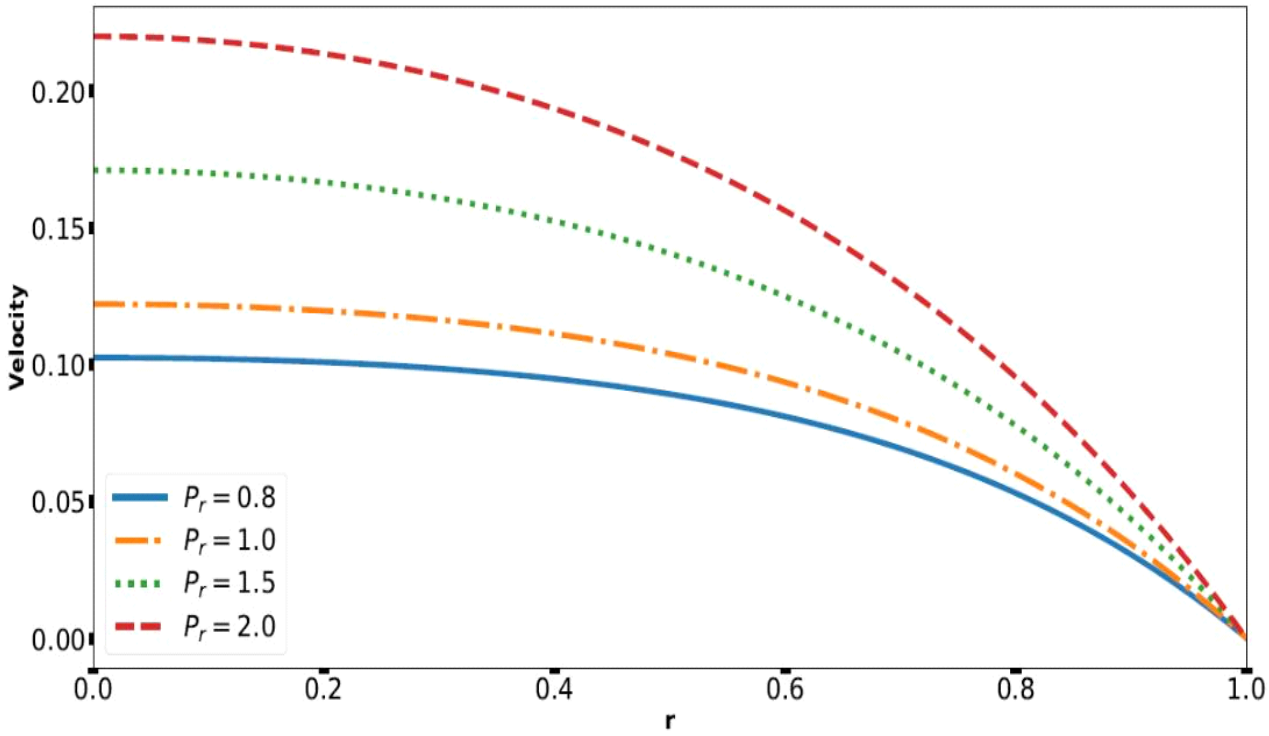

$Gr = 0.05$ **and different values of pressure gradient**

parameter, Pr

Fig. 5 shows an increase in pressure gradient leads to an increase in fluid velocity. Therefore, the result is physically obtainable; it is in agreement with those of [10,23].

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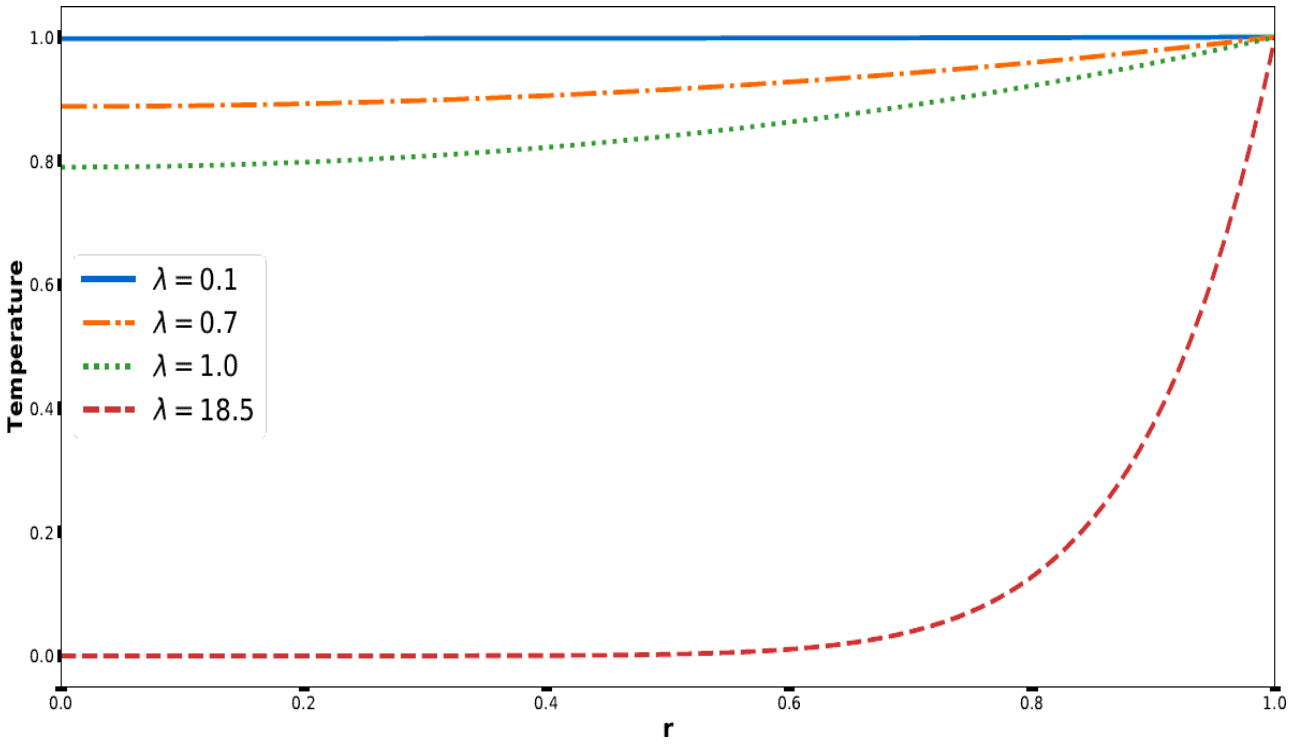

a

Fig. 6. Temperature profiles for with $\theta = 1.0$ and different values of temperature cooling parameter, λ

Fig. 6 shows the variation of fluid temperature with cooling parameter. It is observed that the temperature decreases as the cooling parameter increases. This is quite obvious in our daily lives, if a body is hot and if cooling is applied, the body temperature reduces. This agree with the result of [7].

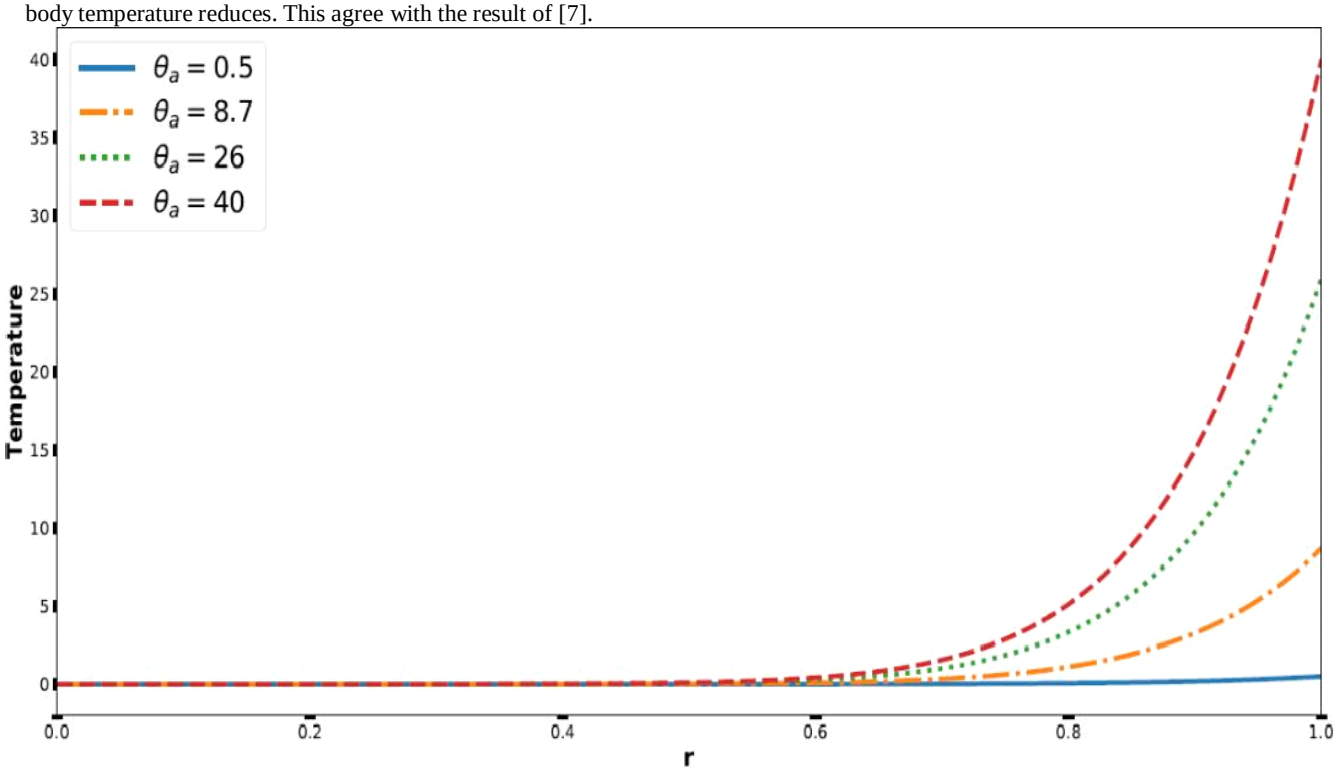

Fig. 7. Temperature profiles for $\lambda = 1.0$ and different values of temperature boundary parameter, θ_a

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It is evident from Fig. 7's temperature variation with temperature boundary condition that fluid temperature rises with increasing boundary parameter.

Fig. 8 illustrates how concentration rises as the injection parameter is increased. This is physically evident since a higher injection value would result in a higher level of pollutant addition to the fluid; Fig. 9 illustrates a similar phenomenon with an increasing boundary condition parameter. It is practical as a result. This is consistent with [7's] outcome.

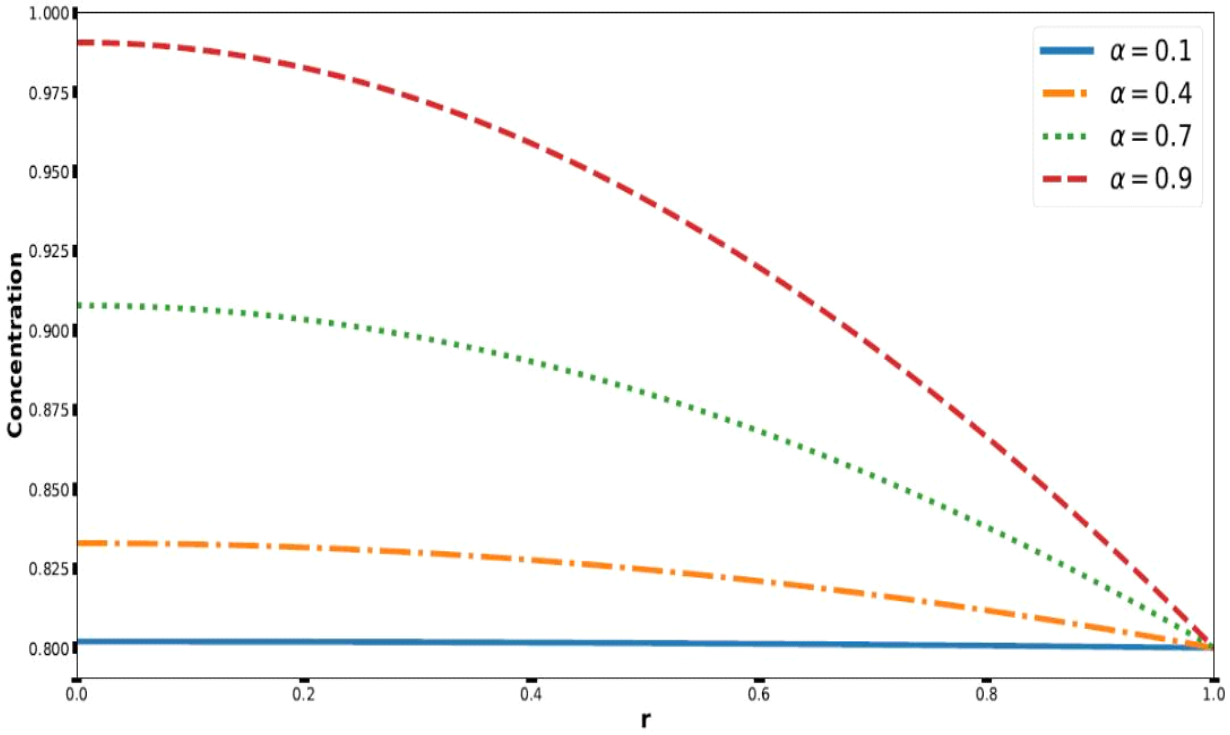

Fig. 8. Concentration profiles for $\varphi_a = 0.1$ and different values of concentration injection parameter,

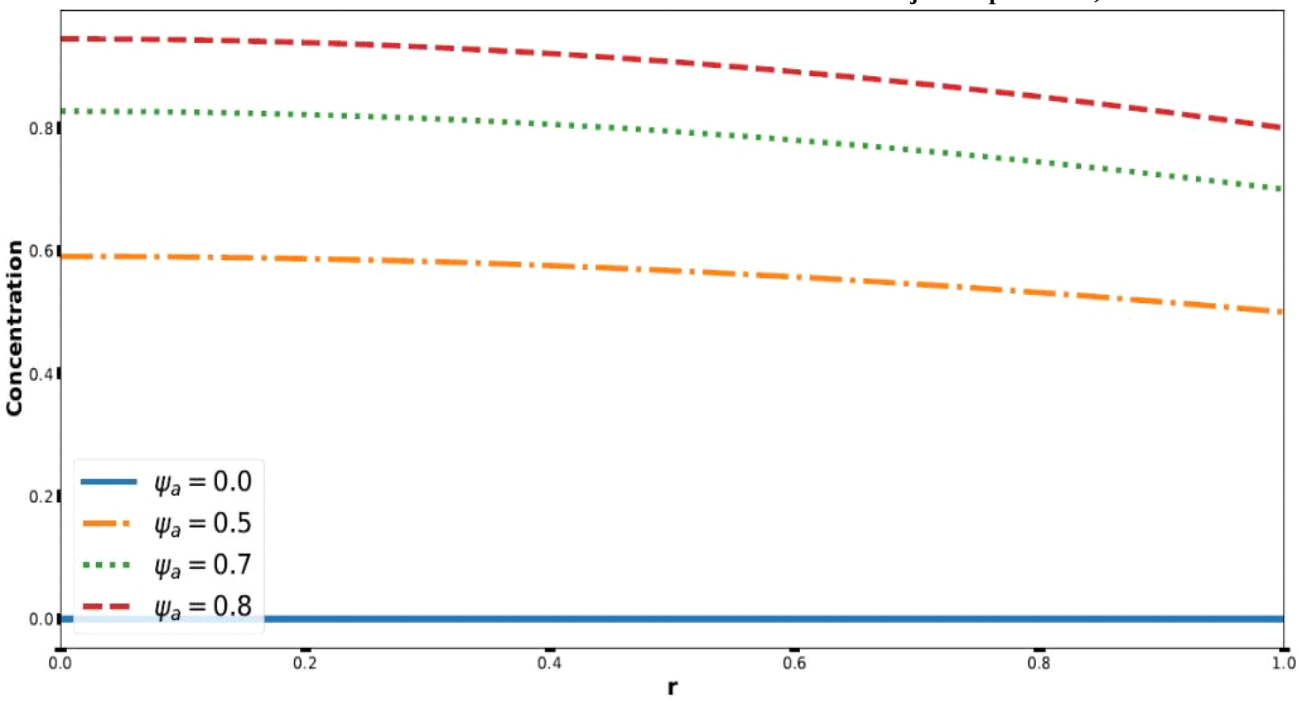

Fig. 9. Concentration profiles for with $\varphi_a = 0.8$ and different values of boundary condition parameter,

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• Conclusions

This work looked into the mass and heat transfer issue in order to simulate non-isothermal flow in a cylindrical channel. Three differential equations are carefully stated as a linked system. Comprehensive analytical answers are provided, followed by a theoretical and graphical verification of the concentration's positive. The graphical results demonstrated that (i) a rise in the cooling parameter lowers the fluid velocity; (ii) a rise in the cooling parameter lowers the temperature; and (iii) a rise in the injection parameter increases the fluid concentration.

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